Let's look at geometric progressions (GP) now. Before I start, let me point out that GMAT is unlikely to give you a statement which looks like this: "If S denotes a geometric progression whose first term is… " GMAT will not test your knowledge of GP (i.e. you don't really need to learn the formulas of the sum of n terms of a GP or sum of infinite terms of a GP etc) though it may give you a sequence which is a geometric progression and ask you questions on it. You will be able to solve the question without using the formulas but recognizing a GP can help you deal with such questions in an efficient manner. That is the reason we are discussing GPs today.

For those of you who are wondering what exactly a GP is, let me begin by giving you the definition.

(I will quote Wikipedia here.)

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the common ratio. For example, the sequence 2, 6, 18, 54, … is a geometric progression with common ratio 3 (each term after the first is obtained by multiplying the previous term by 3). Similarly 10, 5, 5/2, 5/4, … is a geometric sequence with common ratio 1/2.

The first term of a GP is generally denoted by a and the common ratio is denoted by r. So the general form of a GP is: a, ar, ar^2, ar^3, …, ar^(n – 1) (The GP has n terms here.)

Sum of n terms of a GP is given by a*(1 – r^n)/(1 – r).

Sum of all the terms of an infinite GP is given by a/(1 – r).

We generally look at the derivation of the formula to help us understand it better (and hence remember it) but the derivation of this formula is more mathematical and less intuitive so we will not discuss it here. If you are interested in the derivation, check out the Wikipedia link given above. We will directly jump to GMAT relevant questions now and see how they can be solved without the aid of the formulas and how they can be solved with the formulas. First we will look at a simple DS question which deals with GP but doesn't mention it in the question stem. The challenge is to figure out that it is a GP. Once you do, you can solve it in a few moments.

Question 1: In a certain sequence, when you subtract the Mth element from the (M-1)th element, you get twice the Mth element (M is any positive integer). What is the fourth element of this sequence?

1. The first element of the sequence is 1.
2. The third element of the sequence is 1/9.

Solution: The question stem doesn't tell us that the sequence is a geometric progression. It only tells us that t(m-1) – t(m) = 2*t(m)

This tells us that t(m) = t(m-1)/3
Every subsequent term should be a third of the previous term. This means that the sequence is a GP with common ratio = 1/3.

So the GP looks something like this: a, a/3, a/9, a/27 …

Statement 1: The first term of the sequence is 1. Now we know all the terms of the sequence.

1, 1/3, 1/9, 1/27…

The fourth element is 1/27 so this statement alone is sufficient.

Statement 2: The third term of the sequence is $1/9$.

The third term of our GP is $a/9$. If $a/9 = 1/9$, this implies that $a = 1$. Hence the fourth term $= 1/27$. This statement alone is also sufficient to answer the question.

Answer (D).

Now we will look at a trickier question.

Question 2: What is the value of $7 + 6*7 + 6*7^2 + 6*7^3 + 6*7^4 + 6*7^5 + 6*7^6$?

(A) 6^7
(B) 6^9
(C) 7^7
(D) 7^8
(E) 7^9

Solution: First let’s solve the question without using the GP formula.

Method 1:

$S = 7 + 6*7 + 6*7^2 + 6*7^3 + 6*7^4 + 6*7^5 + 6*7^6$

$S = 7*(1 + 6) + 6*7^2 + 6*7^3 + 6*7^4 + 6*7^5 + 6*7^6$ (Take 7 common from the first two terms)

$S = 7^2 + 6*7^2 + 6*7^3 + 6*7^4 + 6*7^5 + 6*7^6$

$S = 7^2 * (1 + 6) + 6*7^3 + 6*7^4 + 6*7^5 + 6*7^6$ (Take 7^2 common from the first two terms)

$S = 7^3 + 6*7^3 + 6*7^4 + 6*7^5 + 6*7^6$

I hope you see where we are going with this. The last step would be:

$S = 7^6 * (1 + 6) = 7^7$

Answer (C)

Method 2:

Except for the first term of the sequence, the rest of the sequence is a GP with first term as $6*7$ and the common ratio as 7.

$S = 7 + 6*7 + 6*7^2 + 6*7^3 + 6*7^4 + 6*7^5 + 6*7^6 = 7 + GP$

There are 6 terms in the GP.

Sum of the GP $= a*(1 – r^n)/(1 – r) = 6*7*(1 – 7^6)/(1 – 7) = 6*7 * (7^6 – 1)/6 = 7^7 – 7$

Substituting this sum back in S, we get

$S = 7 + 7^7 – 7 = 7^7$

Answer (C)

The first method is not difficult. It is just hard to figure out when you are under time pressure. A GP is easy to notice and you know exactly how to handle it. I will not advice you to use one method over the other – both are equally valid and good so use whatever works for you. The only thing is that knowing how to deal with GPs can help save time. It may not be very apparent in this question but I will leave you with a question in which it will be apparent! It is something similar to the 700+ level GMAT prep test question we saw while working on arithmetic progressions. We will discuss this question in detail next week.

Question 3: For every integer n from 1 to 200, inclusive, the n*th* term of a certain sequence is given by (-1)^n*2^(-n). If N is the sum of the first 200 terms in the sequence, then N is

(A) less than -1
(B) between -1 and -1/2
(C) between -1/2 and 0
(D) between 0 and 1/2
(E) greater than 1/2

See you next week!